F.LE.A.4: Exponential Decay

1 Depreciation (the decline in cash value) on a car can be determined by the formula \( V = C(1 - r)^t \), where \( V \) is the value of the car after \( t \) years, \( C \) is the original cost, and \( r \) is the rate of depreciation. If a car’s cost, when new, is $15,000, the rate of depreciation is 30\%, and the value of the car now is $3,000, how old is the car to the nearest tenth of a year?

2 The amount \( A \), in milligrams, of a 10-milligram dose of a drug remaining in the body after \( t \) hours is given by the formula \( A = 10(0.8)^t \). Find, to the nearest tenth of an hour, how long it takes for half of the drug dose to be left in the body.

3 The equation for radioactive decay is \( p = (0.5)^{\frac{t}{H}} \), where \( p \) is the part of a substance with half-life \( H \) remaining radioactive after a period of time, \( t \). A given substance has a half-life of 6,000 years. After \( t \) years, one-fifth of the original sample remains radioactive. Find \( t \), to the nearest thousand years.

4 One of the medical uses of Iodine–131 (I–131), a radioactive isotope of iodine, is to enhance x-ray images. The half-life of I–131 is approximately 8.02 days. A patient is injected with 20 milligrams of I–131. Determine, to the nearest day, the amount of time needed before the amount of I–131 in the patient’s body is approximately 7 milligrams.

5 A radioactive substance has a mass of 140 g at 3 p.m. and 100 g at 8 p.m. Write an equation in the form \( A = A_0 \left( \frac{1}{2} \right)^{\frac{t}{h}} \) that models this situation, where \( h \) is the constant representing the number of hours in the half-life, \( A_0 \) is the initial mass, and \( A \) is the mass \( t \) hours after 3 p.m. Using this equation, solve for \( h \), to the nearest ten thousandth. Determine when the mass of the radioactive substance will be 40 g. Round your answer to the nearest tenth of an hour.